Robert Finn’s career has been largely motivated by problems of fluid mechanics, which lead to mathematical interpretations within the disciplines of Analysis and of Geometry. His initial paper characterized a range of pde’s of geometry and of physics, whose solutions admit no isolated singularities. This essentially nonlinear behavior is an intrinsic property of minimal surfaces, and turns out also to permeate deeply the general studies of gas dynamics and of capillarity.

He continued to introduce new concepts, among them “Equations of Minimal Surface Type” characterized by conformal properties of the equation coefficients. In a very different direction, he studied idealized flow through jets. In both instances he obtained decisive new theorems, and also counterexamples to then “established” literature.

He published with D. Gilbarg a series of papers on compressible flow past obstacles. A particular consequence was that if a subsonic flow exists, then there exists no flow with the same velocity at infinity and which is partly supersonic.

Finn then took up the study of viscous flow past obstacles. He obtained the first characterization of the steady flow of fluid past a 3-D obstacle, and together with D. R. Smith extended the result to the more difficult (in view of the “Stokes Paradox”) 2-D case.

He made some forays into the world of global differential geometry, then in 1969 he and P. Concus initiated a series of papers displaying exotic properties of capillary surfaces, arising from the particular nonlinearity in the defining equation. Of special interest was the behavior at a corner point of opening $2\alpha$ in the horizontal section of a cylindrical capillary tube, in which a finite volume of fluid meets the side walls in a “contact angle” $\gamma$; a crucial new result of the study was that singular behavior occurs whenever $\alpha + \gamma < \pi/2$.

This prediction was experimentally confirmed in real-life configurations, by T. Coburn in a kitchen sink (positive gravity) and by W. Masica in a NASA drop tower (zero gravity).

Finn and E. Giusti proved remarkable a priori regularity properties of any surface $u(x,y)$ of mean curvature $H \equiv 1$ defined over a disk of radius $R > R_0 = 0.5654062332...$. These properties fail if $R < R_0$.

In zero gravity, one finds that even for cylindrical capillary tubes with convex analytic section, there may exist no solution surface simply covering the base; existence criteria were initially known only in limited configurations. Finn introduced a “subsidiary variational problem” within the base domain, and obtained the “Nonexistence-Existence Principle” that the nonexistence of a solution to the subsidiary problem is equivalent to the existence of a smooth solution to the original physical problem. This result includes all other known criteria as special cases.

Jointly with R. Neel, Finn showed that whenever solutions fail to exist in a given domain, there is an idealized singular solution over a subdomain, becoming infinite over a circular arc of determined radius. These solutions are in general not uniquely determined, and in some cases both singular and globally smooth solutions can exist over the same domain.

Three papers by Finn and co-authors addressed a question raised initially by M. Miranda, as to whether for given contact angle $0 < \gamma < \pi/2$, a capillary tube with “small” section raises liquid higher than does one with “large” section. All three papers yielded negative answers, the final one
with both sections concentric disks. P. Concus and V. Brady confirmed the predictions numerically.

During his retirement years, Finn has been studying floating criteria for solid bodies partly immersed in liquids, and the mutual attractions or repulsions of simultaneously floating bodies. He has already authored or co-authored six research papers on the former question, and another six on a special case introduced by Laplace for the latter. These studies led to predictions of rise height and associated forces, for fluid between two parallel partially immersed vertical plates. Some of the predictions were recently confirmed in delicate experiments, by Sung Kwon Cho and his students.